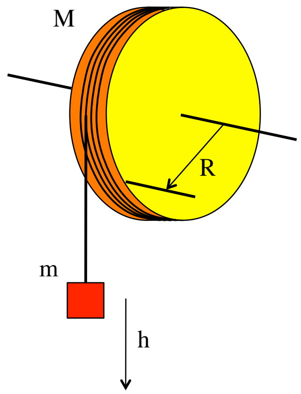
Problem 10.51

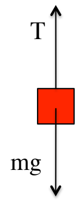
A large, massive pulley (modeled as a disk) has a rope wound around it with a hanging mass attached to the free end. The mass is released from rest and falls 6.00 meters.

PRELIMINARY NOTE: This is a typical AP problem in the sense that a scenario is set up, then you are asked to use everything and the kitchen sink to answer questions about it. Enjoy!

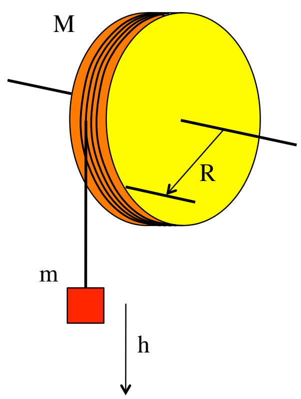
a.) What is the *tension* in the rope as the mass accelerates downward?

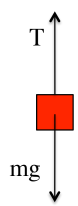


This is a N.S.L., and we will need both the rotational and translational versions to accommodate all the unknowns. Starting with a f.b.d. on the hanging mass, we get:



NOTE: A dumb but common error is to short-cut the process by looking at the f.b.d. and jumping to the conclusion that the tension T = mg. It isn't if there is acceleration in the mix! Noting that the acceleration is downward in the direction I normally define as *negative* (hence the need to unembed the negative sign associated with the "a" term), the process yields:

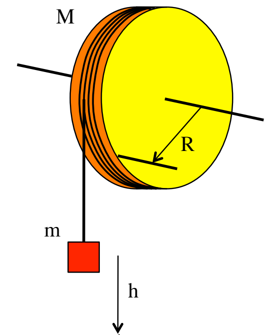


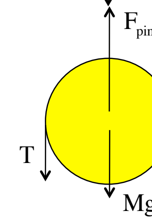


$$\underline{\sum F_y :}$$
$$T - mg = -ma$$
$$\Rightarrow \quad a = \frac{-T + mg}{m}$$
$$\Rightarrow \quad a = \frac{-T}{m} + g$$

Summing the torques:

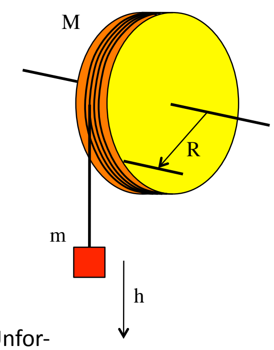
$$\underline{\sum \Gamma_{pin} :}$$
$$TR = I\alpha$$
$$\Rightarrow \quad TR = \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right)$$
$$\Rightarrow \quad TR = \left(\frac{1}{2}MR^2\right)\left(\frac{\left(\frac{-T}{m}+g\right)}{R}\right)$$
$$\Rightarrow \quad T = -\left(\frac{1}{2}M\right)\left(\frac{T}{m}\right) + \left(\frac{1}{2}M\right)g$$
$$\Rightarrow \quad T + \left(\frac{M}{2m}\right)T = \left(\frac{1}{2}M\right)g$$
$$\Rightarrow \quad T = \frac{\left(\frac{1}{2}M\right)g}{1 + \frac{M}{2m}}$$
$$\Rightarrow \quad T = \frac{\left(\frac{1}{2}(3.00\text{ kg})\right)(9.80\text{ m/s}^2)}{1 + \frac{(3.00\text{ kg})}{2(5.10\text{ kg})}}$$
$$= 11.4\text{ N}$$





b.) What is the *acceleration* of the hanging mass?

$$a = \frac{-T}{m} + g$$
$$= \frac{-(11.4\text{ N})}{(5.10\text{ kg})} + (9.80\text{ m/s}^2)$$
$$= 7.56\text{ m/s}^2$$



c.) What is the speed of the mass just before it hits the table 6.00 meters below.

If I had my druthers, I'd use *conservation of energy*. Unfortunately, it is easier to do this using kinematics (energy will be used in *Part d*). Using kinematics:

$$(v_{y,2})^2 = (\overset{0}{\cancel{v_{y,1}}})^2 + 2a_y\Delta y$$
$$\Rightarrow \quad v_{y,2} = \left(2a_y(y_2 - y_1)\right)^{1/2}$$
$$= \left(2(-7.56\text{ m/s}^2)((-6.00\text{ m}) - 0)\right)^{1/2}$$
$$= 9.53\text{ m/s}$$

d.) What is the speed of the mass just before it hits the table 6.00 meters using *conservation of energy*.

$$\sum KE_1^{\,0} + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2^{\,0}$$

$$0 + mgh + 0 = \left[\left(\frac{1}{2}mv_2^2\right) + \left(\frac{1}{2}I_{pully}\omega^2\right)\right]$$

$$0 + mgh + 0 = \left[\left(\frac{1}{2}mv_2^2\right) + \left(\frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_2}{R}\right)^2\right)\right]$$

$$\Rightarrow mgh = \left[\left(\frac{1}{2}mv_2^2\right) + \left(\frac{1}{4}Mv_2^2\right)\right]^{1/2}$$

$$\Rightarrow v^2 = \frac{mgh}{\left(\frac{1}{2}m\right) + \left(\frac{1}{4}M\right)}$$

$$\Rightarrow v = \left[\frac{(5.10 \text{ kg})(9.80 \text{ m/s}^2)(6.00 \text{ m})}{\left(\frac{1}{2}(5.10 \text{ kg})\right) + \left(\frac{1}{4}(3.00 \text{ kg})\right)}\right]^{1/2}$$

$$\Rightarrow v = 9.53 \text{ m/s}$$

IT MATCHES!